INFLUENCE OF THE SPIN ON THE POSITION OF REGGE POLES

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Received 30 November 1962

Recently Gribov and Pomeranchuk, when studying singularities of a partial-wave amplitude as a function of the orbital angular momentum, have discovered condensations of poles in the l-plane [1,2]. First of all we mean the condensations appearing at threshold energy for the production of n particles [1]. The position of these condensations in the complex l-plane is $l = -(3n - 5)/2$. Such a phenomenon exists both in wave mechanics and in relativistic field theory.

In the field theory there are also condensations at negative integer l, at any energy, due to crossed channels [2].

All these condensations restrict the asymptotic behaviour of the amplitude on the momentum transfer, which is in fact the energy in the other channel.

In the present paper an influence of spin on the positions of condensation just described is considered, and the results are briefly discussed.

a. Let us consider for simplicity the scattering of two neutral spinless particles (π-mesons). Particles with spin, e.g. a nucleon pair, may appear in the intermediate state. Near the threshold for pair production the production amplitude is proportional to p^l, where p is the momentum of the produced particles and l is their orbital angular momentum. The value of l depends on the orbital angular momentum j of the π-mesons and the resultant spin of the particles produced. The minimum value of l is $j - 2\sigma$, where σ is the spin of each particle. It is easy to see that this value is never forbidden by parity conservation Reproducing arguments of Gribov and Pomeranchuk [1] one can easily obtain that such a dependence on p leads to condensation of poles at $j = -\frac{1}{2} + 2\sigma$ when $p \to 0$. This condensation appears, of course, also in the scattering amplitude of spinless particles, and in all the amplitudes connected with it by the unitarity condition as well.

Just in the same way every threshold for n-particle production gives rise to the condensation of singularities at $j = -(3n - 5)/2 + n\sigma$, where for simplicity each particle has the same spin σ.

The known elementary particles have a spin not greater than 1. However, if the Coulomb interaction were switched off, the nuclei could have, in principle, arbitrarily large spin. But even if the nuclear spin is limited, it is possible to take a sufficiently large number of nuclei, with spins greater than $\frac{3}{2}$, so that at the appropriate energy poles turn out to be arbitrarily far to the right.

Up to now we have not taken into account the identity of the particles involved. Due to the Pauli principle the minimum value of l in a many fermion system is greater than $j - n\sigma$. As has been pointed out by Pomeranchuk (private communication) one can show that for that reason the condensations of poles which are due to the threshold of such a fermion system can not be placed too far to the right at $n \to \infty$. However in the case of a boson system the identity of particles does not prevent the poles from going infinitely far to the right.

This means that the number of subtractions in the dispersion relation in the momentum transfer increases unlimitedly when the energy goes to infinity. Therefore the scattering amplitude has an essential singularity at $t \to \infty$, $s \to \infty$, and the Mandelstam representation is wrong. Nevertheless, basic results obtained by means of it seem to remain valid. For instance, according to Greenberg and Low [3] we do not need to know the behaviour of the amplitude at large momentum transfer to obtain Froissart's result [4].

b. Let us consider condensations which exist at any energy [2]. Let us consider, for example, the nucleon-antinucleon pair in the intermediate state. The kinematics of $\pi\pi \to N\bar{N}$ reaction has been studied by Frazer and Fulco [5]. Partial-wave amplitudes with definite value of the total angular momentum j can be expressed in terms of $A_j(t)$, $B_{j+1}(t)$ and $B_{j-1}(t)$, where A and B are the usual invariant amplitudes, and $A_l(t)$ and $B_l(t)$ have the following structure:

$$\frac{1}{\pi}\int_{z_0}^{\infty} Q_l(z')\, B^{(1)}(s',t)\, dz' \,. \tag{1}$$

The quantities $A_l(t)/(pk)^l$ and $B_l(t)/pk)^l$ have left and right hand cuts in the t-plane. Generally speaking, the discontinuities on the left-hand cut have poles in l at the negative integers [2]. The residue of the pole at $l = -n-1$ is proportional to the integral:

$$\int_{-z_1(t)}^{z_1(t)} P_n(z)\, b(s,u)\, dz \,, \qquad (2)$$

where $b(s,u)$ is the Mandelstam spectral function. The integration is carried out at constant t over the region where $b(s,u) \neq 0$. If the Mandelstam representation does not hold, $b(s,u)$ should be considered as the discontinuity of $B^{(1)}(s,t)$ at $t < 0$.

As was shown in [2] the poles of the left-hand cut discontinuity give rise to condensation of poles of the amplitude near the negative integers in the l-plane. This is a consequence of the unitarity condition in the t channel. Such a condensation exists at any energy.

Generally speaking, in the case of the reaction $\pi\pi \to N\bar{N}$ the condensations appear at $j = 0$ as well as at negative integral j. These condensations arise also in the π-π scattering amplitude and in all the amplitudes connected with it. The residues of the poles condensing to zero are proportional to j. This corresponds to the fact that the state with $l = j-1$ is switched off at $j = 0$.

The condensation of poles at zero may be tested experimentally. In particular, the NN-scattering at high energy and not too small angles should be determined by this condensation. The present experimental data [6] are not sufficient to establish its presence or absence.

The condensation at zero may not exist if the corresponding integral (2) vanishes for all t. As a matter of fact, only the diagram of fig. 1 contributes to (2) for not very large negative t.

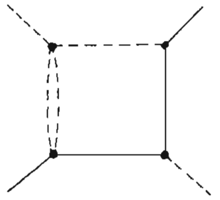

Fig. 1.

Here the condition for the vanishing of (2) is an integral relation between $\pi\pi$ scattering amplitude and the amplitudes A and B of πN scattering. At other t this relation would include the amplitudes of multiple processes.

If we consider now the general case of pair production of particles with spin σ, poles could condense at zero and positive integers up to $j = -1 + 2\sigma$. For $\sigma > 1$ the condensations would be at $j = 2$ or greater. But this would contradict the unitarity condition in the s channel. For $\sigma = 1$ there is no direct contradiction, but it would mean that the diffraction cone is not narrowed. This would not agree with the experimental data [7]. Therefore, one can think that the condensation at $j = 1$ is also absent.

The number of conditions which are necessary to satisfy unitarity increases rapidly with σ. The amplitude for two meson annihilation of a pair of particles with spin σ contains

$$N = \tfrac{1}{2}(2\sigma+1)^2 + \tfrac{1}{4}\left[1 - (-1)^{2\sigma+1}\right]$$

invariant functions. In order to eliminate condensations at positive integer j it is necessary to make integrals of the type (2) vanish. This gives rise to $(N-1)(N-2)/2$ intergral relations.

c. In the preceeding sections we have described some difficulties appearing in the relativistic theory on account of particles with high spin. But elementary particles with spin greater than unity are absent. The nuclei which have such spin, but nuclear amplitudes have anomalous singularities, the structure of which is not sufficiently known at present.

To obtain the results of the first section we assumed only that partial-wave amplitudes may be analytically continued to complex j with threshold behaviour preserved. This assumption seems to be of a very general character. It is not violated with the usual real anomalous singularities.

The assumptions of the second section are of more detailed character. It is possible that if nuclear structure were taken into account the problem would change so much that the elimination of the condensations would not require any supplementary conditions.

It is very interesting to note that the two photon intermediate state does not lead to condensation of poles either at $j = 1$ or at $j = 0$ because of gauge invariance.

I am very grateful to A. A. Anselm, V. N. Gribov, G. S. Danilov, I. T. Dyatlov, I. Ya. Pomeranchuk, V. M. Shekhter and I. M. Shmushkevich for many valuable discusions.

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